

Measures of central tendency

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1. Arithmetic, weighted, geometric and harmonic means and their relation
2. Median, mode and partition values (quartiles, deciles, percentiles) for raw and grouped data

1 Arithmetic, weighted, geometric and harmonic means and their relation

The arithmetic mean shares the total equally, the geometric mean averages ratios and growth rates, the harmonic mean averages rates such as speeds, and for positive values always $AM \geq GM \geq HM$.

Core facts

- **Arithmetic mean (AM)**, written $\bar{x}$:
 - individual series: $\bar{x} = \Sigma x \div n$;
 - discrete series: $\bar{x} = \Sigma fx \div N$, $N = \Sigma f$;
 - continuous series: x = class mid-value, same formula;
 - short-cut (assumed mean A , $d = x - A$): $\bar{x} = A + \Sigma fd \div N$;
 - step deviation ($d' = (x - A)/h$): $\bar{x} = A + (\Sigma fd' \div N) \times h$.
- **Properties of AM:**
 - $\Sigma(x - \bar{x}) = 0$: deviations from the mean add up to zero;
 - $\Sigma(x - A)^2$ is least when $A = \bar{x}$;
 - **combined mean** $\bar{x}_{12} = (n_1\bar{x}_1 + n_2\bar{x}_2) \div (n_1 + n_2)$;
 - change of origin and scale: if $u = (x - A)/h$ then $\bar{x} = A + h \bar{u}$.
- **Weighted AM:** $\bar{x}_w = \Sigma wx \div \Sigma w$. With equal weights it equals the simple AM.
- **Geometric mean (GM)**: the n th root of the product of n values.
 - $GM = (x_1 \times x_2 \times \dots \times x_n)^{(1/n)}$;
 - by logarithms: $GM = \text{antilog}(\Sigma \log x \div n)$; grouped: $\text{antilog}(\Sigma f \log x \div N)$;
 - weighted: $\text{antilog}(\Sigma w \log x \div \Sigma w)$.
- **Harmonic mean (HM)**: the reciprocal of the AM of reciprocals.
 - $HM = n \div \Sigma(1/x)$; grouped: $HM = N \div \Sigma(f/x)$.
- **Relation:** for positive values $AM \geq GM \geq HM$; all three are equal only when every value is the same.
- **Two positive numbers a and b :** $AM = (a + b)/2$, $GM = \sqrt{ab}$, $HM = 2ab/(a + b)$, and $GM^2 = AM \times HM$.
- **Uses:** AM for ordinary averages; **GM** for average **rates of growth**, ratios, percentages and **index numbers**; **HM** for average **speed** over equal distances and average **price** when equal amounts of money are spent.
- GM and HM cannot be used when any value is **zero**; GM is not used with **negative** values.

Example 1. Find the AM of 12, 18, 25, 30 and 15. *Solution.* $\Sigma x = 100$, $n = 5$. **Answer: 20**

Example 2. Find the GM of 4, 9 and 6. *Solution.* Product = 216; cube root of 216. **Answer: 6**

Example 3. Find the HM of 2, 3 and 6. *Solution.* $\Sigma(1/x) = 1/2 + 1/3 + 1/6 = 1$. $HM = 3 \div 1$. **Answer: 3**

Explained

1. AM of a continuous series by step deviation. Choose A as a central mid-value and h as the class width so that d' becomes small whole numbers.

Example 4. Find the mean.

Class	Mid-value x	f	$d' = (x - 25)/10$
0-10	5	6	-2
10-20	15	10	-1
20-30	25	14	0
30-40	35	12	1
40-50	45	8	2

Solution. $fd' = -12, -10, 0, 12, 16$, so $\Sigma fd' = 6$ and $N = 50$. $\bar{x} = 25 + (6 \div 50) \times 10 = 25 + 1.2$. **Answer: 26.2** (direct method: $\Sigma fx = 1,310$; $1,310 \div 50 = 26.2$.)

2. Weighted mean. Use it when items are not equally important (subjects with different credits, commodities in an index).

Example 5. A student scores 60, 75 and 80 in three papers with weights 2, 3 and 5. Find the weighted mean. *Solution.* $\Sigma wx = 120 + 225 + 400 = 745$; $\Sigma w = 10$. **Answer: 74.5** (the simple mean would be 71.67).

3. Combined mean.

Example 6. Section A has 40 students with mean 52; section B has 60 students with mean 47. Find the mean of all 100. *Solution.* $(40 \times 52 + 60 \times 47) \div 100 = (2,080 + 2,820) \div 100$. **Answer: 49**

4. Corrected mean. Correct the total, then divide again.

Example 7. The mean of 50 observations is 36. Later it was found that 43 was wrongly copied as 34. Find the correct mean. *Solution.* Wrong total 1,800. Correct total = $1,800 - 34 + 43 = 1,809$. $1,809 \div 50$. **Answer: 36.18**

5. GM for average growth. Growth rates multiply, so their average is the GM of the growth factors $(1 + r)$, not the AM of the rates.

Example 8. Production rose by 8%, 12% and 25% in three successive years. Find the average annual rate of growth. *Solution.* Factors 1.08, 1.12, 1.25; product = 1.512. $GM = 1.512^{(1/3)} = 1.1478$ (log route: $\Sigma \log = 0.17955$, $\div 3 = 0.05985$, antilog = 1.1478). **Answer: about 14.78% a year** (the AM of the rates, 15%, overstates it).

6. HM for average speed.

Example 9. A car goes from A to B at 60 km/h and returns at 40 km/h. Find the average speed for the round trip. *Solution.* Equal distances, so use HM: $2 \div (1/60 + 1/40) = 2 \div (5/120)$. **Answer: 48 km/h** (not 50).

Example 10. A buyer spends ₹100 on rice at ₹4 per kg and another ₹100 at ₹5 per kg. Find the average price per kg. *Solution.* Equal money spent, so HM: $2 \div (1/4 + 1/5) = 40/9$. **Answer: about ₹4.44 per kg** (₹200 buys $25 + 20 = 45$ kg).

7. The three means side by side.

Example 11. For 2, 3 and 6: $AM = 11/3 = 3.67$, $GM = 36^{1/3} = 3.30$, $HM = 3$. The order $AM > GM > HM$ holds, as it must for unequal positive values.

Mean	Best for	Gives most weight to	Fails when
AM	ordinary totals shared equally	large values	extreme values distort it
GM	ratios, growth rates, index numbers	large values less than the AM does	any value is zero or negative
HM	speeds, rates per unit (equal amounts)	small values	any value is zero

Deeper

1. Why $AM \geq GM \geq HM$ (two numbers). $(\sqrt{a} - \sqrt{b})^2 \geq 0$ gives $a + b \geq 2\sqrt{ab}$, so $AM \geq GM$. Since $HM = GM^2 \div AM$ and $AM \geq GM$, $HM \leq GM$. Equality holds only when $a = b$.

Example 12. The AM of two numbers is 25 and their HM is 16. Find the GM and the numbers. *Solution.* $GM = \sqrt{(25 \times 16)} = 20$. So $a + b = 50$ and $ab = 400$, giving 40 and 10 (check: $HM = 2 \times 400 \div 50 = 16$). **Answer: GM 20; numbers 40 and 10**

2. Grouped GM and HM.

Example 13. Find the AM, GM and HM.

x	5	10	20	40
f	2	3	3	2

Solution. $N = 10$.

- $AM = (10 + 30 + 60 + 80) \div 10 = \mathbf{18}$.
- $\log x = 0.6990, 1.0000, 1.3010, 1.6021$; $\Sigma f \log x = 11.5051$; $\div 10 = 1.15051$; antilog = **14.14** (exactly $\sqrt{200}$).
- $\Sigma(f/x) = 0.4 + 0.3 + 0.15 + 0.05 = 0.9$; $HM = 10 \div 0.9 = \mathbf{11.11}$.

Answer: AM 18, GM 14.14, HM 11.11

3. Weighted HM (unequal amounts). When different amounts are spent or distances differ, weight each rate by the amount or distance.

Example 14. ₹100 is spent on a commodity at ₹4 per kg and ₹200 at ₹5 per kg. Find the average price. *Solution.* Quantity = 25 + 40 = 65 kg for ₹300. Weighted HM = $300 \div (100/4 + 200/5)$. **Answer: about ₹4.62 per kg**

Example 15. A cyclist covers 30 km at 30 km/h and then 90 km at 45 km/h. Find the average speed.

Solution. Time = 1 h + 2 h = 3 h for 120 km. **Answer: 40 km/h**

4. Missing frequency.

Example 16. The mean of the distribution is 25. Find p.

x	10	20	30	40
f	5	p	10	5

Solution. $(50 + 20p + 300 + 200) \div (20 + p) = 25$ gives $550 + 20p = 500 + 25p$, so $p = 10$. **Answer: 10**

5. Useful properties of GM.

- $\log GM$ = mean of the logs, so GM suits data that grow by multiplication.
- GM of the ratios of two series = ratio of their GMs; this is why the GM index satisfies the **time reversal test** (see index numbers).
- Multiplying every value by k multiplies the AM, GM and HM by k ; adding k changes the AM by k but not the GM or HM by k .

6. Merits and limits of the AM (textbook list). Rigidly defined, based on all items, capable of algebraic treatment, least affected by sampling fluctuations; but distorted by extreme values, cannot be found for open-end classes without assumptions, and cannot be located by inspection or graph.

Common traps

✗ "Average growth rate = AM of the yearly rates." ✓ Use the **GM of the factors $(1 + r)$** , then subtract 1.

✗ "Average speed over equal distances = $(60 + 40)/2 = 50$." ✓ Use the **HM: 48 km/h**.

✗ " $HM \geq GM \geq AM$." ✓ **$AM \geq GM \geq HM$** for positive values.

✗ " $GM^2 = AM + HM$." ✓ For two numbers **$GM^2 = AM \times HM$** .

✗ " $\sum(x - \bar{x})^2$ is zero." ✓ **$\sum(x - \bar{x}) = 0$** ; $\sum(x - \bar{x})^2$ is the **minimum** sum of squares.

✗ "In step deviation, $\bar{x} = A + \sum fd' \div N$." ✓ Multiply by the class width: **$\bar{x} = A + (\sum fd' \div N) \times h$** .

✗ "Combined mean = average of the two means." ✓ Weight by sizes: **$(n_1\bar{x}_1 + n_2\bar{x}_2) \div (n_1 + n_2)$** .

Quick revision

- $AM = \sum fx \div N$; step deviation $A + (\sum fd' / N) \times h$.
- $\sum(x - \bar{x}) = 0$; $\sum(x - A)^2$ least at $A = \bar{x}$.
- Weighted AM = $\sum wx \div \sum w$; combined mean weights by n .
- $GM = (\text{product})^{1/n} = \text{antilog } (\sum f \log x \div N)$.
- $HM = N \div \sum(f/x)$.
- $AM \geq GM \geq HM$; equal only if all values equal.
- Two numbers: $GM^2 = AM \times HM$.
- GM: growth rates, ratios, index numbers.
- HM: speeds over equal distances, prices with equal spend.
- Zero value: no GM or HM.

2 Median, mode and partition values (quartiles, deciles, percentiles) for raw and grouped data

The median splits ordered data into two equal halves, quartiles, deciles and percentiles split it into 4, 10 and 100 equal parts, and the mode is the value that occurs most often; for grouped data each is found by interpolation inside its class.

Core facts

- **Median, quartiles, deciles, percentiles** are **positional averages** (partition values); data must be **arranged in order** first.
- **Number of dividing points:** quartiles **3** (Q_1, Q_2, Q_3), deciles **9**, percentiles **99**.
- **Equalities:** **Median** = $Q_2 = D_5 = P_{50}$; $Q_1 = P_{25}$; $Q_3 = P_{75}$; $D_1 = P_{10}$.
- **Individual and discrete series** (textbook convention):
 - median = size of the $((N + 1)/2)$ th item;
 - $Q_1 = ((N + 1)/4)$ th item, $Q_3 = (3(N + 1)/4)$ th item;
 - $D_i = (i(N + 1)/10)$ th item, $P_i = (i(N + 1)/100)$ th item;
 - in a discrete series, find the first cumulative frequency (cf) that equals or exceeds the position.
- **Continuous series** (use $N/2, N/4 \dots$ without the +1):
 - **Median** = $l + [(N/2 - c) \div f] \times h$;
 - $Q_1 = l + [(N/4 - c) \div f] \times h$, $Q_3 = l + [(3N/4 - c) \div f] \times h$;
 - $D_i = l + [(iN/10 - c) \div f] \times h$, $P_i = l + [(iN/100 - c) \div f] \times h$;
 - l = lower limit of the class containing the value, c = cf of the class **before** it, f = its frequency, h = class width.
- **Mode (Z):** the value with the highest frequency.
 - continuous series: $Z = l + [(f_1 - f_0) \div (2f_1 - f_0 - f_2)] \times h$, also written $l + [\Delta_1 \div (\Delta_1 + \Delta_2)] \times h$ with $\Delta_1 = f_1 - f_0$, $\Delta_2 = f_1 - f_2$;
 - f_1 = modal class frequency, f_0 = class before, f_2 = class after.
- **Empirical relation (Karl Pearson)**, moderately skewed data: **Mode = 3 Median - 2 Mean**.
- **Graphs:** the median is the x-value where the **less-than and more-than ogives** cross (or where $N/2$ meets the less-than ogive); the mode is located from the **histogram**.
- Median and mode can be found for **open-end classes**; the mean cannot without assumptions.
- $\Sigma|x - A|$ is least when **A = median**.

Example 1. Find the median of 15, 22, 8, 30, 19, 27, 11. *Solution.* Sorted: 8, 11, 15, 19, 22, 27, 30. $N = 7$, the 4th item. **Answer: 19**

Example 2. Find the median of 5, 12, 7, 20, 15, 9. *Solution.* Sorted: 5, 7, 9, 12, 15, 20. The 3.5th item: $(9 + 12)/2$. **Answer: 10.5**

Explained

1. Quartiles of raw data.

Example 3. For 3, 5, 7, 8, 10, 12, 14, 15, 18, 20, 22 find Q_1 , the median and Q_3 . *Solution.* $N = 11$ (already sorted). $Q_1 = (12/4)^{\text{th}} = 3^{\text{rd}}$ item; median = 6th item; $Q_3 = (36/4)^{\text{th}} = 9^{\text{th}}$ item. **Answer: $Q_1 = 7$, median = 12, $Q_3 = 18$**

2. Discrete series.

Example 4. Find the median, Q_1 , Q_3 and mode.

x	10	20	30	40	50
f	4	7	12	9	8
cf	4	11	23	32	40

Solution. $N = 40$.

- Median: $(41/2) = 20.5^{\text{th}}$ item; first cf ≥ 20.5 is 23 $\rightarrow$ **30**.
- Q_1 : $(41/4) = 10.25^{\text{th}}$ item; first cf ≥ 10.25 is 11 $\rightarrow$ **20**.
- Q_3 : $3 \times 41/4 = 30.75^{\text{th}}$ item; first cf ≥ 30.75 is 32 $\rightarrow$ **40**.
- Mode: highest frequency 12 $\rightarrow$ **30**.

3. Continuous series: one table, every measure.

Example 5.

Class	f	cf
0-10	5	5
10-20	8	13
20-30	15	28
30-40	20	48
40-50	12	60
50-60	10	70

$N = 70$ (the mean of this table is 33).

Measure	Position	Class (l, c, f)	Value
Median	$N/2 = 35$	30-40 (30, 28, 20)	$30 + (7/20) \times 10 = \mathbf{33.5}$
Q_1	$N/4 = 17.5$	20-30 (20, 13, 15)	$20 + (4.5/15) \times 10 = \mathbf{23}$
Q_3	$3N/4 = 52.5$	40-50 (40, 48, 12)	$40 + (4.5/12) \times 10 = \mathbf{43.75}$
D_7	$7N/10 = 49$	40-50 (40, 48, 12)	$40 + (1/12) \times 10 = \mathbf{40.83}$
P_{90}	$90N/100 = 63$	50-60 (50, 60, 10)	$50 + (3/10) \times 10 = \mathbf{53}$

Mode: modal class 30–40 ($f_1 = 20$, $f_0 = 15$, $f_2 = 12$). $Z = 30 + [5 \div (40 - 27)] \times 10 = 30 + 3.85$. **Answer: about 33.85** Check: $3 \times 33.5 - 2 \times 33 = 34.5$, close to 33.85, as expected for nearly symmetric data.

4. Empirical relation.

Example 6. In a moderately skewed distribution the mean is 52 and the mode is 46. Find the median.

Solution. Median = $(2 \text{ Mean} + \text{Mode}) \div 3 = (104 + 46) \div 3$. **Answer: 50**

5. Mode when the modal class is the first class. Take the frequency of the missing class before it as 0.

Example 7. Classes 0–20, 20–40, 40–60 with frequencies 30, 20, 10. Find the mode. *Solution.* $f_1 = 30$, $f_0 = 0$, $f_2 = 20$. $Z = 0 + [30 \div (60 - 0 - 20)] \times 20$. **Answer: 15**

Feature	Median	Mode
Type	positional	most frequent value
Open-end classes	can be found	can be found
Best for	skewed data (incomes, wages)	most common size, fashion, qualitative data
Weakness	ignores the size of extreme items	may be ill-defined or bimodal

Deeper

1. Inclusive classes. Convert to exclusive (true) class limits before using any formula: subtract half the gap (0.5 here) from lower limits and add it to upper limits.

Example 8. Find the median and mode.

Class	10–19	20–29	30–39	40–49
f	6	14	10	5

Solution. True classes 9.5–19.5, 19.5–29.5, ... $N = 35$, $N/2 = 17.5$, median class 19.5–29.5 ($c = 6$, $f = 14$).

- Median = $19.5 + (11.5/14) \times 10 = \mathbf{27.71}$.
- Mode = $19.5 + [8 \div (28 - 6 - 10)] \times 10 = \mathbf{26.17}$.

Using 20 instead of 19.5 as l is the usual mistake and gives a wrong answer.

2. Missing frequency from a given median.

Example 9. The median of the distribution is 27. Find p .

Class	0–10	10–20	20–30	30–40	40–50
f	5	p	20	15	5

Solution. $N = 45 + p$. The median 27 lies in 20–30, so $c = 5 + p$, $f = 20$. $27 = 20 + [((45 + p)/2 - (5 + p)) \div 20] \times 10$ gives $14 = (35 - p)/2$, so $p = 7$ ($N = 52$; check: $20 + (26 - 12)/20 \times 10 = 27$). **Answer: 7**

3. Ill-defined mode. When two or more values share the highest frequency, or the highest frequency is near the ends, the mode is located by the **grouping method** (frequencies added in twos and threes, the value occurring most often in the grouping table is the mode) or, failing that, by the empirical relation.

4. Unequal class widths. Before applying the mode formula, adjust frequencies to a common width (frequency density), since a wider class collects more items only because it is wider.

5. Graphical location.

- Less-than ogive: plot cf against upper limits; read the x-value at height $N/2$ for the median, $N/4$ for Q_1 , $3N/4$ for Q_3 .
- Both ogives cross at the median.
- Histogram: join the top-right corner of the modal bar to the top-right corner of the bar before it, and the top-left corner of the modal bar to the top-left corner of the bar after it; drop a perpendicular from where these lines cross to the x-axis; its foot is the mode.

6. Uses of partition values. Quartiles give the **quartile deviation** $(Q_3 - Q_1)/2$ and **Bowley's skewness**; percentiles are used for ranks in competitive exams and growth charts (the 90th percentile means 90% scored at or below that value).

Common traps

✗ "Median of 15, 22, 8, 30, 19 is 8, the middle value as written." ✓ **Arrange in order first:** median **19**.

✗ "In the grouped median formula, c is the cf of the median class." ✓ c is the cf of the class **before** it.

✗ "Mode = 3 Mean - 2 Median." ✓ **Mode = 3 Median - 2 Mean.**

✗ "There are 4 quartiles and 10 deciles." ✓ **3 quartiles, 9 deciles, 99 percentiles.**

✗ "For a continuous series the median class is found at $(N + 1)/2$." ✓ Use **$N/2$** for continuous series.

✗ "With inclusive classes 20–29, $I = 20$." ✓ Use the true limit **19.5**.

✗ "The median is located where the less-than ogive meets the x-axis." ✓ It is where the **two ogives intersect**.

Quick revision

- Arrange data first: positional averages.
- Raw/discrete: median $(N + 1)/2$ th, Q_1 $(N + 1)/4$ th, Q_3 $3(N + 1)/4$ th item.
- Continuous: $I + [(position - c)/f] \times h$, position $N/2$, $N/4$, $3N/4$, $iN/10$, $iN/100$.
- Median = $Q_2 = D_5 = P_{50}$.
- 3 quartiles, 9 deciles, 99 percentiles.
- Mode = $I + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$.
- Mode = 3 Median - 2 Mean.
- Ogives cross at the median; histogram gives the mode.
- Inclusive classes: convert to true limits.

- Median and mode work with open-end classes.