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STATISTICS

# Time series and index numbers

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Chapter 1 · Statistics

# Time series and index numbers

3 TOPICS

## 1.1 Time series: components, measuring trend (graphic, semi-averages, moving averages, least squares) and seasonal variation

*A time series is data arranged in order of time; its movement is split into trend (T), seasonal (S), cyclical (C) and irregular (I) parts, and the trend is measured by the graphic, semi-averages, moving averages or least squares method.*

### CORE FACTS

- **Time series:** values of a variable recorded at successive points or periods of time (yearly output, monthly prices, daily sales).
- **Four components:**
  1. **Secular trend (T):** smooth, long-term rise or fall (population growth, falling death rate).
  2. **Seasonal variation (S):** regular, repeating pattern **within a year** (weekly, monthly, quarterly); caused by climate, festivals, customs.
  3. **Cyclical variation (C):** up-and-down swings over **more than a year** (business cycle: prosperity, recession, depression, recovery); period not fixed.
  4. **Irregular (random, erratic) variation (I):** unpredictable effects of floods, wars, strikes, epidemics.
- **Additive model:** $Y = T + S + C + I$ (components independent). **Multiplicative model:** $Y = T \times S \times C \times I$ (components related; most used).
- **Methods of measuring trend:** (1) freehand / graphic, (2) semi-averages, (3) moving averages, (4) least squares (most used, gives a mathematical equation).
- **Moving average period** should equal the **period of the cycle**; an **odd** period is centred directly, an **even** period must be **centred** again by a 2-item moving average.
- **Least squares straight line:** $Y = a + bX$. Normal equations: $\Sigma Y = na + b\Sigma X$, $\Sigma XY = a\Sigma X + b\Sigma X^2$.
- If X is measured from the middle so that $\Sigma X = 0$: $a = \Sigma Y / n$, $b = \Sigma XY / \Sigma X^2$.
- Least squares line passes through the **means**; $\Sigma(Y - Yc) = 0$ and $\Sigma(Y - Yc)^2$ **is minimum**.

- **Methods of measuring seasonal variation:** (1) simple averages, (2) ratio to trend, (3) ratio to moving average (most widely used), (4) link relatives.
- **Seasonal indices** total **400** for quarters and **1,200** for months (average 100).

**Example 1.** Sales of umbrellas rise every June–August. Which component is this? *Solution.* It repeats every year within the year. **Answer: seasonal variation**

**Example 2.** Fit a straight line by least squares to 7 years when $\Sigma Y = 126$, $\Sigma XY = 84$, $\Sigma X^2 = 28$ (X from the middle year). Find a and b. *Solution.* a = 126 ÷ 7 = 18; b = 84 ÷ 28 = 3. **Answer: Y = 18 + 3X**

**1. Graphic (freehand) method.** Plot the data and draw a smooth line through the middle of the fluctuations. Simple and flexible, but **subjective**: two people draw different lines, and it cannot be relied on for forecasting.

**2. Method of semi-averages.** Divide the series into two equal halves (drop the middle year if the number of years is odd), find the average of each half, plot each average against the **middle of its half**, and join the two points.

**Example 3.** Output (thousand tonnes): 2018: 40, 2019: 44, 2020: 42, 2021: 48, 2022: 52, 2023: 50, 2024: 56, 2025: 60. Find the trend by semi-averages and estimate 2026. *Solution.*

- First half (2018–21): (40 + 44 + 42 + 48) ÷ 4 = 174 ÷ 4 = 43.5, placed at mid-2019 and 2020 (2019.5).
- Second half (2022–25): (52 + 50 + 56 + 60) ÷ 4 = 218 ÷ 4 = 54.5, placed at 2023.5.
- Annual change = (54.5 − 43.5) ÷ 4 = 2.75.
- 2026 is 2.5 years after 2023.5: 54.5 + 2.5 × 2.75 = 61.375.

**Answer: trend rises 2.75 a year; 2026 estimate about 61.4 thousand tonnes**

Merits: objective, easy. Limits: assumes a **straight-line** trend; the arithmetic mean is affected by extreme values.

**3. Method of moving averages.** Average the first m values, then drop the first and add the next, and so on; place each average at the **centre** of its period.

**Example 4.** Find the 3-yearly moving averages: 2019: 20, 2020: 24, 2021: 22, 2022: 26, 2023: 30, 2024: 28, 2025: 34. *Solution.*

| Year | Value | 3-year total | 3-year MA |
|---|---|---|---|
| 2019 | 20 | – | – |

| Year | Value | 3-year total | 3-year MA |
|---|---|---|---|
| 2020 | 24 | 66 | 22.00 |
| 2021 | 22 | 72 | 24.00 |
| 2022 | 26 | 78 | 26.00 |
| 2023 | 30 | 84 | 28.00 |
| 2024 | 28 | 92 | 30.67 |
| 2025 | 34 | – | – |

**Answer: 22, 24, 26, 28, 30.67 for 2020–2024** (no trend values for the first and last year)

**Example 5.** Using the output data of Example 3, find the 4-yearly centred moving averages. *Solution.*

- 4-year totals: 174, 186, 192, 206, 218, centred at 2019.5, 2020.5, 2021.5, 2022.5, 2023.5.
- Centre them: add pairs of totals and divide by 8.
  - 2020: (174 + 186) ÷ 8 = 45.00
  - 2021: (186 + 192) ÷ 8 = 47.25
  - 2022: (192 + 206) ÷ 8 = 49.75
  - 2023: (206 + 218) ÷ 8 = 53.00

**Answer: 45, 47.25, 49.75, 53 for 2020–2023** (two years lost at each end)

Merits: simple, flexible, removes cyclical swings if the period equals the cycle. Limits: trend values are **lost at both ends**; no equation, so no forecasting; choice of period matters.

**4. Method of least squares (straight line).** Choose the line $Y = a + bX$ that makes the **sum of squared deviations** of actual values from the line a minimum. Take the middle year as origin so $\Sigma X = 0$.

**Example 6.** Fit a straight-line trend and estimate 2027: 2019: 8, 2020: 13, 2021: 16, 2022: 17, 2023: 21, 2024: 24, 2025: 27. *Solution.*

- Origin 2022, X = −3, −2, −1, 0, 1, 2, 3; $\Sigma X = 0$, $\Sigma X^2 = 9 + 4 + 1 + 0 + 1 + 4 + 9 = 28$.
- $\Sigma Y = 126$. $\Sigma XY = -24 - 26 - 16 + 0 + 21 + 48 + 81 = 84$.
- $a = 126 \div 7 = 18$; $b = 84 \div 28 = 3$. Trend: **$Y = 18 + 3X$** (origin 2022, X in years, Y in the data's units).
- Trend values 2019–2025: 9, 12, 15, 18, 21, 24, 27.

- 2027: X = 5, Y = 18 + 15 = 33.

**Answer: Y = 18 + 3X; 2027 estimate = 33**

Merits: objective, gives trend for **every** year and a formula for forecasting. Limits: heavy arithmetic; adding one new year changes all values; the chosen type of curve (line, parabola) must fit the data.

**5. Seasonal variation by simple averages.** Average each season over the years; divide each seasonal average by the grand average of the seasonal averages; multiply by 100.

**Example 7.** Quarterly sales (Rs lakh) for three years. Find the seasonal indices.

| Year | Q1 | Q2 | Q3 | Q4 |
|---|---|---|---|---|
| 2023 | 60 | 48 | 52 | 72 |
| 2024 | 64 | 52 | 58 | 78 |
| 2025 | 68 | 56 | 61 | 84 |

*Solution.*

- Quarter averages: Q1 = 192 ÷ 3 = 64; Q2 = 156 ÷ 3 = 52; Q3 = 171 ÷ 3 = 57; Q4 = 234 ÷ 3 = 78.
- Grand average = (64 + 52 + 57 + 78) ÷ 4 = 251 ÷ 4 = 62.75.
- Indices: Q1 = 64 ÷ 62.75 × 100 = 101.99; Q2 = 82.87; Q3 = 90.84; Q4 = 124.30. Total = 400.

**Answer: 101.99, 82.87, 90.84, 124.30 (Q4 is the peak season)**

The simple average method assumes there is **no trend**; when the data rise over time (as here), later quarters are overstated, so the other methods are preferred.

### Deeper

**1. Even number of years in least squares.** With an even n there is no middle year. Take the origin midway between the two middle years and measure X in **half-years**: X = −5, −3, −1, 1, 3, 5 for six years. Then b is the change per **half-year**; double it for the yearly change.

**Example 8.** Fit a straight line: 2020: 30, 2021: 34, 2022: 37, 2023: 43, 2024: 47, 2025: 49. Estimate 2026. *Solution.*

- Origin between 2022 and 2023; X = −5, −3, −1, 1, 3, 5; $\Sigma X^2 = 25 + 9 + 1 + 1 + 9 + 25 = 70$.
- ΣY = 240; ΣXY = −150 − 102 − 37 + 43 + 141 + 245 = 140.
- a = 240 ÷ 6 = 40; b = 140 ÷ 70 = 2 (per half-year), so 4 per year.

- Y = 40 + 2X. For 2026, X = 7: Y = 40 + 14 = 54.

**Answer: Y = 40 + 2X (X in half-years); 2026 estimate = 54**

**2. Second-degree (parabolic) trend.** $Y = a + bX + cX^2$. Normal equations:

- $\Sigma Y = na + b\Sigma X + c\Sigma X^2$
- $\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$
- $\Sigma X^2Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$

With $\Sigma X = \Sigma X^3 = 0$: $\Sigma Y = na + c\Sigma X^2$, $\Sigma XY = b\Sigma X^2$, $\Sigma X^2Y = a\Sigma X^2 + c\Sigma X^4$.

**Example 9.** Fit a parabola to 5 years: 6, 7, 10, 15, 22 (X = −2, −1, 0, 1, 2). *Solution.*

- $\Sigma X^2 = 10$, $\Sigma X^4 = 16 + 1 + 0 + 1 + 16 = 34$, $\Sigma Y = 60$, $\Sigma XY = -12 - 7 + 0 + 15 + 44 = 40$, $\Sigma X^2Y = 24 + 7 + 0 + 15 + 88 = 134$.
- $b = 40 \div 10 = 4$.
- $60 = 5a + 10c$ and $134 = 10a + 34c$. Doubling the first: $120 = 10a + 20c$; subtracting, $14 = 14c$, so $c = 1$, $a = 10$.

**Answer: $Y = 10 + 4X + X^2$**

**3. Other seasonal methods.**

- **Ratio to trend:** fit a trend (least squares), express each original value as a % of its trend value, average these percentages season by season, adjust to total 400 (or 1,200). Removes trend; cycles are not removed.
- **Ratio to moving average:** compute a 12-month (or 4-quarter) **centred** moving average (it contains T and C), divide each original value by it (leaving S × I), average season-wise to remove I, then adjust. **Most satisfactory and widely used.**
- **Link relatives (Pearson's method):** link relative = current season ÷ previous season × 100; average them, chain them, correct for trend, adjust. Laborious.

**4. Deseasonalising.** Under the multiplicative model, deseasonalised value = actual value ÷ seasonal index × 100. Example: Q4 sales of 84 with index 124.30 → 84 ÷ 1.2430 = 67.6 (about).

**5. Choosing the moving average period.** If the data have a regular cycle of k years, a k-year moving average removes it completely; a wrong period leaves part of the cycle in the trend.

| Method | Equation? | Ends lost? | Objective? |
|---|---|---|---|
| Freehand | no | no | no |

| Method | Equation? | Ends lost? | Objective? |
|---|---|---|---|
| Semi-averages | line only | no | yes |
| Moving averages | no | yes | yes |
| Least squares | yes | no | yes |

**COMMON TRAPS**

- ✗ "Business cycles repeat within a year." ✓ That is **seasonal** variation; cycles last **more than a year**.
- ✗ "A 4-year moving average is placed against the 2nd year." ✓ It falls **between** two years; it must be **centred** again.
- ✗ "Moving averages give trend values for every year." ✓ Values are **lost at both ends**.
- ✗ "With 6 years, X = −3 to 2 makes ΣX = 0." ✓ Use **−5, −3, −1, 1, 3, 5** (half-year units), then double b for the yearly change.
- ✗ "Seasonal indices of four quarters total 100." ✓ They total **400** (1,200 for months).
- ✗ "Least squares minimises the sum of deviations." ✓ It minimises the sum of **squared** deviations; the plain sum is **zero**.
- ✗ "The simple average method works for data with a strong trend." ✓ It assumes **no trend**; use ratio to moving average.

## 1.2 Index numbers: types and methods (Laspeyres, Paasche, Fisher, Marshall-Edgeworth), tests of adequacy

*An index number measures the relative change in a group of related variables (prices, quantities, values) between a base period (= 100) and a current period; the weighted formulas of Laspeyres (base-year quantities), Paasche (current-year quantities) and Fisher (their geometric mean) are judged by the unit, time reversal, factor reversal and circular tests.*

**CORE FACTS**

- Index numbers are called **economic barometers**. Base period index = **100**.
- Notation: $p_0$, $q_0$ = base-year price and quantity; $p_1$, $q_1$ = current-year price and quantity. $P_{01}$ = price index for year 1 on base 0.
- **Types:** price index, quantity (volume) index, value index ($\Sigma p_1q_1 \div \Sigma p_0q_0 \times 100$).

- **Simple aggregative:** $P_{01} = \Sigma p_1 \div \Sigma p_0 \times 100$.
- **Price relative:** $R = p_1 \div p_0 \times 100$. **Simple average of relatives:** $\Sigma R \div n$ (AM) or antilog $(\Sigma \log R \div n)$ (GM).
- **Laspeyres:** $P_{01} = \Sigma p_1 q_0 \div \Sigma p_0 q_0 \times 100$ (weights = **base-year** quantities).
- **Paasche:** $P_{01} = \Sigma p_1 q_1 \div \Sigma p_0 q_1 \times 100$ (weights = **current-year** quantities).
- **Fisher:** $P_{01} = \sqrt{(\text{Laspeyres} \times \text{Paasche})}$ — the **geometric mean** of L and P; called the **ideal index**.
- **Marshall-Edgeworth:** $P_{01} = \Sigma p_1(q_0 + q_1) \div \Sigma p_0(q_0 + q_1) \times 100$ (weights = sum, i.e. average, of both years' quantities).
- **Dorbish-Bowley:** (L + P) ÷ 2 — the **arithmetic mean** of L and P.
- **Kelly (fixed weight):** $\Sigma p_1 q \div \Sigma p_0 q \times 100$, q = fixed quantities of some chosen period.
- **Quantity indices:** swap p and q, e.g. Laspeyres $Q_{01} = \Sigma q_1 p_0 \div \Sigma q_0 p_0 \times 100$.
- **Tests of adequacy (Fisher):**
  1. **Unit test:** index unaffected by units of measurement. Failed only by the **simple aggregative** index.
  2. **Time reversal test:** $P_{01} \times P_{10} = 1$. Passed by **Fisher**, **Marshall-Edgeworth**, simple aggregative, simple GM of relatives, Kelly. Failed by **Laspeyres, Paasche**, simple AM of relatives.
  3. **Factor reversal test:** $P_{01} \times Q_{01} = \Sigma p_1 q_1 \div \Sigma p_0 q_0$ (value ratio). Passed **only by Fisher**.
  4. **Circular test:** $P_{01} \times P_{12} \times P_{20} = 1$. Passed by the **simple GM of relatives**, the **simple aggregative** and **Kelly's fixed-weight** index; **not** by Fisher.
- Fisher is "ideal" because it passes the **time reversal and factor reversal** tests (not because it passes all tests).
- **Geometric mean** is theoretically the best average for index numbers.

**Example 1.** Laspeyres index = 120 and Paasche index = 118.75. Find Fisher's index. *Solution.* $\sqrt{(120 \times 118.75)} = \sqrt{14{,}250}$. **Answer: 119.37 (about)**

**Example 2.** Which index satisfies the factor reversal test? (a) Laspeyres (b) Paasche (c) Marshall-Edgeworth (d) Fisher *Solution.* Only Fisher's formula gives $P_{01} \times Q_{01}$ = value ratio. **Answer: (d)**

### 1. Problems in constructing an index number.

- **Purpose:** decides items, prices and weights (a CPI for workers uses retail prices; WPI uses wholesale prices).
- **Base period:** a **normal** year, free from wars, floods, booms; not too distant. Fixed base or chain base.
- **Selection of items:** representative, standard quality, enough in number.
- **Price quotations:** wholesale or retail, from representative markets.
- **Choice of average:** AM is easy; **GM** gives equal weight to equal ratios and passes the time reversal test.
- **Weights:** quantity weights (aggregative) or value weights (relatives); weighting is needed because items differ in importance.

### 2. Simple (unweighted) indices and the weighted formulas: one data set.

| Item | $p_0, q_0$ | $p_1, q_1$ | Relative $p_1/p_0$ |
|---|---|---|---|
| A | 10, 4 | 12, 5 | 120 |
| B | 8, 10 | 10, 8 | 125 |
| C | 5, 6 | 6, 10 | 120 |
| D | 20, 2 | 22, 3 | 110 |

Products:

| Item | $p_0q_0$ | $p_1q_0$ | $p_0q_1, p_1q_1$ |
|---|---|---|---|
| A | 40 | 48 | 50, 60 |
| B | 80 | 100 | 64, 80 |
| C | 30 | 36 | 50, 60 |
| D | 40 | 44 | 60, 66 |
| Total | 190 | 228 | 224, 266 |

**Example 3.** Find the simple aggregative index and the simple AM of relatives. *Solution.* $\Sigma p_1 = 12 + 10 + 6 + 22 = 50$; $\Sigma p_0 = 10 + 8 + 5 + 20 = 43$. Simple aggregative $= 50 \div 43 \times 100 = 116.28$. AM of relatives $= (120 + 125 + 120 + 110) \div 4 = 475 \div 4 = 118.75$. **Answer: 116.28 and 118.75**

**Example 4.** Find the Laspeyres, Paasche, Fisher, Marshall-Edgeworth and Dorbish-Bowley price indices. *Solution.*

- Laspeyres = 228 ÷ 190 × 100 = **120.00**
- Paasche = 266 ÷ 224 × 100 = **118.75**
- Fisher = √(120 × 118.75) = **119.37**
- Marshall-Edgeworth = (228 + 266) ÷ (190 + 224) × 100 = 494 ÷ 414 × 100 = **119.32**
- Dorbish-Bowley = (120 + 118.75) ÷ 2 = **119.375**

**Answer: L 120, P 118.75, F 119.37, M-E 119.32, D-B 119.38**

**3. Weighted average of relatives.** $P_{01} = \Sigma(R \times W) \div \Sigma W$. With value weights $W = p_0q_0$ it gives exactly the Laspeyres index: (120 × 40 + 125 × 80 + 120 × 30 + 110 × 40) ÷ 190 = 22,800 ÷ 190 = 120.

**4. Fixed base and chain base.**

- **Fixed base:** every year compared with one base year.
- **Chain base:** each year compared with the **previous** year. Link relative = current value ÷ previous year's value × 100; chain index = link relative × previous chain index ÷ 100.

**Example 5.** Link relatives of three years are 110, 120 and 90 (first year's base = 100). Find the chain indices. *Solution.* Year 1: 110. Year 2: 120 × 110 ÷ 100 = 132. Year 3: 90 × 132 ÷ 100 = 118.8.
**Answer: 110, 132, 118.8**

| Point | Fixed base | Chain base |
|---|---|---|
| Compares with | one base year | previous year |
| New items, weights | hard to add | easy to add |
| Long-run comparison | direct | via chaining |
| Error | does not build up | can accumulate |

**5. Uses of index numbers.** Measure changes in price level and the value of money; guide policy (wages, dearness allowance, monetary policy); deflate money values into real values; show trends in production, trade and business; compare places and periods.

**Deeper**

**1. Checking the tests with the data of Example 4.**

- **Time reversal for Laspeyres:** $P_{01} = 228 \div 190 = 1.2$; $P_{10}$ (year 1 as base, its quantities $q_1$ as

weights) = $\Sigma p_0q_1 \div \Sigma p_1q_1 = 224 \div 266$. Product = $1.2 \times 224 \div 266 = 1.0105 \neq 1$. **Fails.**

- **Time reversal for Fisher:** $P_{01} \times P_{10} = \sqrt{(228/190 \times 266/224)} \times \sqrt{(224/266 \times 190/228)} = 1$. **Passes.**
- **Factor reversal for Fisher:** quantity indices are Laspeyres Q = 224 ÷ 190 = 117.89, Paasche Q = 266 ÷ 228 = 116.67, Fisher Q = √(117.89 × 116.67) = 117.28. $P_{01} \times Q_{01} = 1.1937 \times 1.1728 = 1.40 = 266 \div 190$. **Passes.**

**2. Proof sketch of factor reversal for Fisher.** $P_{01} \times Q_{01} = \sqrt{(\Sigma p_1q_0/\Sigma p_0q_0 \times \Sigma p_1q_1/\Sigma p_0q_1)} \times \sqrt{(\Sigma q_1p_0/\Sigma q_0p_0 \times \Sigma q_1p_1/\Sigma q_0p_1)} = \sqrt{((\Sigma p_1q_1)^2 \div (\Sigma p_0q_0)^2)} = \Sigma p_1q_1 \div \Sigma p_0q_0$.

**3. Bias.** Laspeyres uses old quantities and ignores consumers shifting away from goods whose prices rise, so it tends to **overstate** price rises (upward bias); Paasche tends to **understate** them (downward bias). Fisher lies **between** them. This holds when prices and quantities move in opposite directions; it is not a mathematical law, and the order can reverse.

**4. Fisher's limitations.** Needs current-year quantities every year (costly; that is why official CPIs and WPIs use the Laspeyres type), is harder to compute and to explain, and fails the circular test.

| Index | Unit | Time reversal | Factor reversal |
|---|---|---|---|
| Simple aggregative | fails | passes | fails |
| Laspeyres / Paasche | passes | fails | fails |
| Marshall-Edgeworth | passes | passes | fails |
| Fisher | passes | passes | passes |

Circular test: passed by the simple aggregative, simple GM of relatives and Kelly's fixed-weight index; failed by all four weighted indices above.

**Example 6.** If $\Sigma p_0q_0 = 200$, $\Sigma p_1q_0 = 250$, $\Sigma p_0q_1 = 240$ and $\Sigma p_1q_1 = 300$, find Laspeyres, Paasche and Fisher. *Solution.* L = 250 ÷ 200 × 100 = 125; P = 300 ÷ 240 × 100 = 125; F = √(125 × 125) = 125. **Answer: all 125** (when L = P, Fisher equals both)

**COMMON TRAPS**

- ✗ "Laspeyres uses current-year quantities." ✓ **Laspeyres = base-year** ($q_0$); **Paasche = current-year** ($q_1$).
- ✗ "Fisher's index is the arithmetic mean of L and P." ✓ It is the **geometric mean**; the AM is **Dorbish-Bowley**.
- ✗ "Fisher's index passes every test." ✓ It **fails the circular test**; it is ideal for passing **time reversal and factor reversal**.
- ✗ "Marshall-Edgeworth passes the factor reversal test." ✓ It passes **time reversal** only; factor reversal is **Fisher alone**.
- ✗ "Time reversal test: $P_{01} + P_{10} = 1$." ✓ The **product** $P_{01} \times P_{10} = 1$ (without the factor 100).
- ✗ "Simple aggregative index passes the unit test." ✓ It **fails** it: changing kg to quintal changes the result.
- ✗ "Base-year index = 1 or 0." ✓ Base-year index = **100**.

## 1.3 Base shifting, splicing and deflating index numbers; cost of living index and limitations

*Base shifting re-expresses an index on a new base year, splicing joins an old and a new series into one continuous series, deflating turns money values into real values by dividing by a price index, and the cost of living (consumer price) index measures the change in the cost of a fixed basket bought by a particular class of people.*

**CORE FACTS**

- **Base shifting:** new index = old index of the year ÷ old index of the **new base year** × 100.
- **Splicing** needs an **overlapping year** that appears in both series.
  - **Forward** (old series continued on its old base): new-series index × old-series index of the overlap year ÷ 100.
  - **Backward** (old series brought onto the new base): old-series index × 100 ÷ old-series index of the overlap year.
- **Deflating:** real value = money value ÷ price index × 100. **Real wage** = money wage ÷ CPI × 100.

- **Purchasing power of money** = 100 ÷ price index (in rupees of the base year): it moves **opposite** to the price index.
- **Cost of living index (CLI)** = **consumer price index**: measures the change in the cost of buying the **same goods and services** for a specified class (industrial workers, farm labourers, urban employees). It measures changes in **prices**, not in the standard of living.
- Based on a **family budget enquiry**; major groups: **food, clothing, fuel and light, house rent, miscellaneous**; uses **retail** prices.
- **Two methods:**
  1. **Aggregate expenditure method:** CLI = $\Sigma p_1 q_0 \div \Sigma p_0 q_0 \times 100$ (Laspeyres).
  2. **Family budget method:** CLI = $\Sigma RW \div \Sigma W$, $R = p_1/p_0 \times 100$, $W = p_0 q_0$ (base-year expenditure). Gives the **same** result as method 1.
- **Uses of the CLI:** fixing **dearness allowance** and wages, deflating incomes, measuring purchasing power, price policy.
- **India (2026):** CPI (Rural, Urban, Combined) by **NSO, MoSPI**, new series **base 2024** from **February 2026**; **CPI-IW** by the **Labour Bureau**, base **2016 = 100**, used for DA of central government employees.

**Example 1.** An index (2018 = 100) is 150 in 2022 and 180 in 2024. Shift the base to 2022 and give 2024. *Solution.* 180 ÷ 150 × 100. **Answer: 120**

**Example 2.** The CPI rises from 100 to 125. What happens to the purchasing power of a rupee? *Solution.* 100 ÷ 125 = 0.80. **Answer: it falls to 80 paise of base-year value (a 20% fall)**

**1. Base shifting.** Needed when the base year is too old, or to compare two series with different bases.

**Example 3.** Shift the base of this index from 2018 to 2022.

| Year | Index (2018 = 100) | Index (2022 = 100) |
|---|---|---|
| 2018 | 100 | 66.67 |
| 2019 | 110 | 73.33 |
| 2020 | 120 | 80.00 |
| 2021 | 135 | 90.00 |

| Year | Index (2018 = 100) | Index (2022 = 100) |
|---|---|---|
| 2022 | 150 | 100.00 |
| 2023 | 165 | 110.00 |
| 2024 | 180 | 120.00 |

*Solution.* Divide each old index by 150 (the old index of 2022) and multiply by 100, e.g. 2019: 110 ÷ 150 × 100 = 73.33. **Answer: as in the last column**

Strictly, the correct way is to recompute the index with new-base prices and weights; dividing by the base-year index is an approximation that is exact only for indices that pass the **circular test** (such as fixed-weight aggregative indices).

### 2. Splicing.

**Example 4.** An old series (2015 = 100) ends with 160 in 2020. A new series (2020 = 100) gives 2021: 105, 2022: 112, 2023: 120. Splice the new series to the old base. *Solution.* Multiply each new index by 160 ÷ 100 = 1.6. 2021: 105 × 1.6 = 168.0; 2022: 112 × 1.6 = 179.2; 2023: 120 × 1.6 = 192.0. **Answer: 168.0, 179.2, 192.0 (base 2015)**

**Example 5.** Splice the old series backward onto the 2020 base: 2016: 110, 2017: 118, 2018: 130, 2019: 144, 2020: 160 (2015 = 100). *Solution.* Multiply each by 100 ÷ 160. 2016: 68.75; 2017: 73.75; 2018: 81.25; 2019: 90.00; 2020: 100. **Answer: 68.75, 73.75, 81.25, 90, 100 (base 2020)**

Official series use the same idea, called a **linking factor**, when a CPI or WPI moves to a new base.

### 3. Deflating.

**Example 6.** Find the real wages at base-year prices.

| Year | Money wage (Rs) | CPI | Real wage (Rs) |
|---|---|---|---|
| 2021 | 24,000 | 125 | 19,200 |
| 2022 | 26,400 | 132 | 20,000 |
| 2023 | 28,800 | 144 | 20,000 |
| 2024 | 31,500 | 150 | 21,000 |

*Solution.* Real wage = money wage ÷ CPI × 100, e.g. 2021: 24,000 ÷ 125 × 100 = 19,200. Money wages rose 31.25% (24,000 to 31,500) but real wages rose only 9.375% (19,200 to 21,000); in 2023 the money rise was eaten up by prices. **Answer: as in the last column**

### 4. Cost of living index: family budget method.

**Example 7.** Find the CLI.

| Group | Weight (% of spending) | Price relative |
|---|---|---|
| Food | 40 | 150 |
| Clothing | 10 | 125 |
| Fuel and light | 10 | 140 |
| House rent | 20 | 120 |
| Miscellaneous | 20 | 130 |

*Solution.* ΣRW = 40 × 150 + 10 × 125 + 10 × 140 + 20 × 120 + 20 × 130 = 6,000 + 1,250 + 1,400 + 2,400 + 2,600 = 13,650. ΣW = 100. CLI = 13,650 ÷ 100. **Answer: 136.5**

**Example 8.** With the CLI of Example 7, a worker earned Rs 20,000 in the base year. What income keeps his standard of living? What is the real value of Rs 28,000 now? *Solution.* Needed income = 20,000 × 136.5 ÷ 100 = 27,300 (so a DA of Rs 7,300). Real value = 28,000 ÷ 136.5 × 100 = 20,512.82. **Answer: Rs 27,300; about Rs 20,513 at base prices**

### 5. Cost of living index: aggregate expenditure method.

**Example 9.** A family's base-year basket: rice 100 kg, dal 10 kg, oil 5 litres, sugar 8 kg. Base prices (Rs) 40, 100, 120, 40; current prices 52, 130, 150, 45. Find the CLI. *Solution.* $\Sigma p_0q_0$ = 4,000 + 1,000 + 600 + 320 = 5,920. $\Sigma p_1q_0$ = 5,200 + 1,300 + 750 + 360 = 7,610. CLI = 7,610 ÷ 5,920 × 100. **Answer: 128.55 (about)**

**6. Steps in constructing a cost of living index.** Decide the class of people and the area → conduct a **family budget enquiry** to find what and how much they buy → choose items in each group → collect **retail** prices from local markets → fix weights (expenditure shares) → compute by either method.

## Deeper

### 1. Limitations of index numbers.

- Only **approximate** indicators: based on samples of items, markets and families.
- **Quality changes** and **new products** are hard to handle; a fixed basket goes out of date, so the base must be revised.
- Different formulas give **different results** from the same data (Examples in STA-TI-02).

- Each index suits only its **purpose**: a CPI for industrial workers does not measure the cost of living of farmers or of the rich.
- A CLI shows changes in the **cost** of a fixed basket, not changes in **living standards**; consumers substitute cheaper goods, which a Laspeyres basket ignores (upward bias).
- **International and long-period comparisons** are weak because habits, goods and weights differ.
- Can be **manipulated** by choosing a favourable base year, items or weights.

**2. Why base years are revised in India.** Consumption shifts away from food towards services and transport, so old weights overstate food. The CPI of NSO moved from base 2012 to **base 2024** in February 2026, using weights from the Household Consumption Expenditure Survey 2023-24 with a lower food weight; the WPI moved from 2011-12 to **base 2022-23** in June 2026 alongside a new Producer Price Index (see ECO-IN-04). Linking factors splice the old and new series.

**3. Deflating national income.** Real GDP = nominal GDP ÷ GDP deflator × 100; growth in real terms is roughly nominal growth minus inflation. Using the wrong index (e.g. a CPI for a production series) gives misleading real values.

**Example 10.** Nominal income is Rs 1,20,000 when the price index is 160 (base 100). What is the real income? If real income is to stay at Rs 80,000, what nominal income is needed at index 160? *Solution.* 1,20,000 ÷ 160 × 100 = 75,000. Needed: 80,000 × 160 ÷ 100 = 1,28,000. **Answer: Rs 75,000; Rs 1,28,000**

**COMMON TRAPS**

- ✗ "To shift the base, subtract the new base's index from each value." ✓ **Divide** by the new base year's old index and **multiply by 100**.
- ✗ "Splicing needs no common year." ✓ It needs an **overlapping year** present in both series.
- ✗ "Real wage = money wage × price index ÷ 100." ✓ **Divide**: money wage ÷ price index × 100.
- ✗ "CLI measures changes in the standard of living." ✓ It measures changes in the **cost** of a **fixed** basket.
- ✗ "The family budget method and the aggregate expenditure method give different answers." ✓ With base-year weights they give the **same** answer.
- ✗ "CLI uses wholesale prices." ✓ It uses **retail** prices.
- ✗ "Price index up from 100 to 125 means purchasing power falls 25%." ✓ It falls to 100/125 = 0.80, a **20%** fall.

# Quick revision

READ THIS THE DAY BEFORE THE EXAM

## 1.1 Time series: components, measuring trend (graphic, semi-averages, moving averages, least squares) and seasonal variation

- Components: trend, seasonal (within a year), cyclical (over a year), irregular.
- Additive $Y = T + S + C + I$; multiplicative $Y = T \times S \times C \times I$.
- Trend methods: freehand, semi-averages, moving averages, least squares.
- Semi-averages: two halves, average at the middle of each half.
- Even moving-average period: centre it again.
- Least squares with $\Sigma X = 0$: $a = \Sigma Y/n$, $b = \Sigma XY/\Sigma X^2$.
- Even n: X = ..., −3, −1, 1, 3, ...; yearly change = 2b.
- Seasonal methods: simple averages, ratio to trend, ratio to moving average (best), link relatives.
- Seasonal indices total 400 (quarters) or 1,200 (months).
- Deseasonalised = actual ÷ seasonal index × 100.

## 1.2 Index numbers: types and methods (Laspeyres, Paasche, Fisher, Marshall-Edgeworth), tests of adequacy

- Laspeyres = $\Sigma p_1q_0/\Sigma p_0q_0 \times 100$ (base quantities).
- Paasche = $\Sigma p_1q_1/\Sigma p_0q_1 \times 100$ (current quantities).
- Fisher = $\sqrt{(L \times P)}$, ideal index.
- Marshall-Edgeworth = $\Sigma p_1(q_0 + q_1)/\Sigma p_0(q_0 + q_1) \times 100$.
- Dorbish-Bowley = $(L + P)/2$.
- Unit test: fails only simple aggregative.
- Time reversal: $P_{01} \times P_{10} = 1$ (Fisher, M-E pass; L, P fail).
- Factor reversal: $P_{01} \times Q_{01} = \Sigma p_1q_1/\Sigma p_0q_0$ (Fisher only).
- Circular test: simple aggregative, simple GM and Kelly pass; Fisher fails.
- Chain index = link relative × previous chain index ÷ 100.

## 1.3 Base shifting, splicing and deflating index numbers; cost of living index and limitations

- Base shift: old index ÷ old index of new base × 100.
- Splicing: needs an overlap year; forward × (old overlap ÷ 100), backward × (100 ÷ old overlap).
- Real value = money value ÷ price index × 100.
- Purchasing power of money = 100 ÷ price index.
- CLI = consumer price index for a class of people; retail prices; family budget enquiry.
- Aggregate expenditure: $\Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$.
- Family budget: $\Sigma RW / \Sigma W$ with $W = p_0 q_0$ (same answer).
- Income to keep living standard = base income × CLI ÷ 100.
- Limits: approximate, quality changes, outdated basket, purpose-specific, formula-dependent.
- CPI base 2024 (Feb 2026, NSO); CPI-IW base 2016 (Labour Bureau, DA).

# Practice questions

20 QUESTIONS · ANSWERS AT THE END

1. Match List I (example) with List II (component of time series):

List I:

A. Steady rise in India's population over decades

B. Higher sale of umbrellas every monsoon

C. Alternating boom and slump in business every few years

D. Fall in factory output due to a sudden strike

List II:

1. Irregular variation

2. Cyclical variation

3. Seasonal variation

4. Secular trend

Select the correct answer using the code given below.

a. A-4, B-2, C-3, D-1

b. A-1, B-3, C-2, D-4

c. A-3, B-4, C-2, D-1

d. A-4, B-3, C-2, D-1

2. Consider the following statements about India's official price indices (as of 2026):

1. The all-India Consumer Price Index (Rural, Urban and Combined) is compiled by the National Statistics Office (NSO), MoSPI, and its current series has base year 2024 = 100.

2. The Wholesale Price Index is compiled by the Office of the Economic Adviser, DPIIT, and its current series has base year 2022-23.

3. The Consumer Price Index for Industrial Workers (CPI-IW) is compiled by the NSO with base year 2024 = 100.

Which of the statements given above are correct?

a. 2 and 3 only

b. 1 and 3 only

c. 1 and 2 only

d. 1, 2 and 3

3. Consider the following statements about the link relative method:

1. The link relative of the first season cannot be computed for the first year of the data.

2. Link relatives are obtained by dividing each figure by its centred moving average.

Which of the statements given above is/are correct?

a. 1 only

b. 2 only

c. Both 1 and 2

d. Neither 1 nor 2

4. Which of the following pairs of index and formula (× 100) is NOT correctly matched?

a. Laspeyres – Σp1q0 / Σp0q0

b. Paasche – Σp1q1 / Σp0q1

c. Marshall–Edgeworth – Σp1(q0 + q1) / Σp0(q0 + q1)

d. Dorbish–Bowley – √(Laspeyres × Paasche)

5. The values of a series for 2016 to 2021 are 30, 34, 38, 36, 40 and 44. Using a 4-yearly centred moving average, the trend value for 2018 is

a. 34.5

b. 35.75

c. 37

d. 38.25

6. For a group of commodities, Laspeyres' quantity index is 110 and the value index (Σp1q1/ Σp0q0 × 100) is 165. Paasche's price index is

a. 55

b. 66.67

c. 150

d. 181.5

7. Which of the following is NOT generally regarded as a use of time series analysis?

a. Understanding the past behaviour of a variable

b. Measuring the degree of association between two different variables at one point of time

c. Forecasting future values for planning

d. Comparing actual performance with expected trend and seasonal values

8. Match List I (formula, × 100) with List II (index):

List I:

A. Σp1q0 / Σp0q0

B. Σq1p0 / Σq0p0

C. Σp1q1 / Σp0q0

D. Σp1q1 / Σp0q1

List II:

1. Value index

2. Laspeyres' price index

3. Paasche's price index

4. Laspeyres' quantity index

Choose the correct answer.

a. A-3, B-4, C-1, D-2

b. A-2, B-1, C-4, D-3

c. A-4, B-2, C-1, D-3

d. A-2, B-4, C-1, D-3

9. A series for seven years reads 12, 15, 18, 14, 20, 22, 19. Match List I with List II.

List I: A. 3-yearly moving average for year 2 B. 3-yearly moving average for year 3 C. First 4-yearly moving average before centring D. 4-yearly centred moving average for year 3

List II: 1. 15.75 2. 15 3. 14.75 4. 15.67

a. A-2, B-4, C-3, D-1

b. A-2, B-1, C-3, D-4

c. A-2, B-4, C-1, D-3

d. A-4, B-2, C-3, D-1

10. Assertion (A): A chain base index is preferable when the main interest is in comparing a year's prices with those of the immediately preceding year.

Reason (R): Each link relative compares a year with the year just before it.

Choose the correct answer.

a. Both A and R are true, and R is the correct explanation of A

b. Both A and R are true, but R is not the correct explanation of A

c. A is true, but R is false

d. A is false, but R is true

11. Consider the following statements:

1. Seasonal variations have a fixed period, so they can be predicted fairly accurately.

2. Cyclical variations have a fixed period of 12 months.

Which of the statements given above is/are correct?

a. 1 only

b. 2 only

c. Both 1 and 2

d. Neither 1 nor 2

**12.** Base-year prices of two commodities are ₹2 and ₹5, current-year prices ₹3 and ₹6, base-year quantities 10 and 8, and current-year quantities 10 and 12. What is the Marshall-Edgeworth price index?

a. 130 b. 127.5

c. 128.75 d. 128.57

**13.** Consider the following statements:

1. The additive model Y = T + S + C + I assumes that the four components are independent of one another.
2. In the multiplicative model Y = T × S × C × I, the trend is in original units while S, C and I are expressed as relatives (ratios or indices).

Which of the statements given above is/are correct?

a. 1 only b. 2 only

c. Both 1 and 2 d. Neither 1 nor 2

**14.** Consider the following reasons:

1. The old base year is too distant to be relevant.
2. Two index series with different base years have to be compared.
3. Money values have to be converted into real values.

Which of the above are valid reasons for shifting the base of an index series?

a. 1 and 2 only b. 2 and 3 only

c. 1 and 3 only d. 1, 2 and 3

**15.** Under the additive model, the preliminary seasonal effects of the four quarters are +18, −10, +6 and −22; they must be adjusted so that they add up to zero. The quarterly trend is Yc = 300 + 4X, with X in quarters and X = 0 at the first quarter of year 1. What is the forecast for the third quarter of year 4?

a. 360 b. 362

c. 364 d. 368

16. In a cost of living index by the family budget method, the groups food, rent, clothing, fuel and others have weights 50, 20, 10, 10 and 10 and current price indices 180, 150, 200, 120 and 140. A family earned 6,000 rupees a month in the base period. What are the cost of living index and the income needed now to keep the same standard of living?

a. 158 and 9,480 rupees
b. 166 and 3,960 rupees
c. 180 and 10,800 rupees
d. 166 and 9,960 rupees

17. Under the multiplicative model, the trend value of a quarter is 200 units, the seasonal index is 110, the cyclical relative is 95 and the irregular relative is 100. The original value of the series is

a. 190
b. 205
c. 209
d. 220

18. Two goods have base-year values (p0q0) of ₹42 and ₹58 and price relatives of 150 and 100. Paasche's price index for the same data is 100. Fisher's ideal price index is

a. 110
b. 110.5
c. 121
d. 125

19. The quarterly trend of a firm's sales is Yc = 50 + 2X, with origin at the first quarter of 2024 and X in quarters. Under the additive model the seasonal effect of the fourth quarter is −5. The forecast for the fourth quarter of 2025 is

a. 55
b. 59
c. 60.8
d. 64

20. Consider the following possible uses of a consumer price index:
1. Fixing dearness allowance
2. Deflating money wages to find real wages
3. Measuring the purchasing power of money
4. Measuring the correlation between two variables
Which of the above are uses of a consumer price index?

a. 1 and 2 only
b. 1, 2 and 3 only
c. 2, 3 and 4 only
d. 1, 2, 3 and 4

**Answers:** 1-(d) 2-(c) 3-(a) 4-(d) 5-(b) 6-(c) 7-(b) 8-(d) 9-(a) 10-(a) 11-(a) 12-(d) 13-(c) 14-(a) 15-(c) 16-(d) 17-(c) 18-(a) 19-(b) 20-(b)